Classification of ordinary differential equation

Determine the type, order, and degree of the following differential equation. Additionally, indicate which methods can be used to solve it:

$$(x^3 - y^3)\,dx - 3xy^2\,dy = 0$$

Solution

The order of a differential equation is determined by the highest-order derivative present in the equation. To identify the order, we can rewrite the equation in terms of $\frac{dy}{dx}$:

$$(x^3 - y^3) - 3xy^2\frac{dy}{dx} = 0$$

Here, the derivative present is $\frac{dy}{dx}$, which is of the first order. Therefore, the equation is first-order.

The degree of a differential equation is the exponent of the highest-order derivative after clearing radicals and fractions with respect to the derivatives. In this case, $\frac{dy}{dx}$ appears to the power of 1, and there are no roots or fractional powers involved. Therefore, the degree of the equation is one.

The equation is homogeneous because all terms are of degree 3 in x and y.

Let us verify if the equation is **exact**. Let:

$$M(x, y) = x^3 - y^3, \quad N(x, y) = -3xy^2$$

The differential equation is in the form:

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

To be exact, the following condition must hold:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

We compute the partial derivatives:

- $\dfrac{\partial M}{\partial y} = -3y^2$
- $\dfrac{\partial N}{\partial x} = -3y^2$

Since $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$, the equation is exact.